# Tractive rolling contact on a viscoelastic multi-layered half-space

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This paper presents a semi-analytical modelling of the contact on a multi-layered half-space with viscoelastic behaviour. To carry out this modelling, three main points have been taken into account. First, the calculation of influence coefficients for a multi-layered half-space, then the use of Kalker's theory for elastic tractive rolling contact with Conjugate Gradient Method (CGM) and FFT algorithms, and finally an Elastic/Viscoelastic correspondence allows to turn the elastic solution to a viscoelastic one.

Keywords: tractive rolling, multi-layered half-space, viscoelastic contact

# 1. Introduction

The rolling contact has various applications in the mechanics field and outside. One of them concerns rolling engines such as trains, cars, trucks etc. In civil engineering for example, it is important to study the rolling contact between tyres and the road.

In order to study the rolling contact between the train and the rails, the first studies led to a distinction between free rolling contact and tractive rolling contact. The latter differs from the former in that it takes into account the coefficient of friction between the two bodies in contact. Recently, Manyo et al. [1] used Kalker's theory coupled with semi-analytical methods to obtain results for tractive rolling contact for different contact geometries. We apply this approach to a multi-layered half-space (Figure 1); coupled with an elastic/viscoelastic correspondence as in Wallace et al. [2] for a viscoelastic bilayer.



Figure 1: Unit pressure P and unit shear Q applied on a viscoelastic multi-layered half-space. This scheme is used to determine the influence coefficients.  $J_i(t)$  represents the creep function,  $E_i$  and  $v_i$  the elastic constants of each layer *i*.

### 2. Formulation and numerical implementation

### 2.1. Elastic tractive rolling contact

The contact algorithm iterates between the normal tangential problems (Panagiotopoulos process). The normal problem is classically solved with a semianalytical method, while the tangential problem is solved using the Kalker's theory.

## 2.2. Multi-layered modelling

To find the influence coefficients of the multi-layered half-space, we use the Fourier Transform of the Papkovich-Neuber potentials and the adequate boundary conditions as in Yu [3] for example. Then the numerical inverse FT is used to find the influence coefficients.

# 2.3. Viscoelastic modelling

The Elastic/Viscoelastic correspondence can be found in Wallace et al. [2]. It says that a viscoelastic material is an elastic one at the every step with in addition a history to take into account. Thus, we can write for example:

 $u_{z}^{1}(x, y, t) = \int_{0}^{t} J_{1}(t - \xi) \left[ \tilde{G}^{1}\left(x, y, \frac{J_{z}(t)}{J_{1}(t)} \right) * \frac{\partial p(x, y, \xi)}{\partial \xi} \right] d\xi$ (1) where  $\bar{G}^{1}$  represents the Green function, \* convolution, an  $u_{z}^{1}$  is the surface normal displacement.

#### 3. Results and Discussion

The previous modelling allows solving the threedimensional tractive rolling on a multi-layered halfspace. Figure 2 shows the normalized von Mises stress in the middle plane for a full slip contact of a sphere on a tri-layer body. Note that the full-slip case is a limit case of the tractive rolling contact.



Figure 2: Normalized Von Mises stress in the middle plane of an elastic tri- layered half-space for a full-slip (friction f=0.25) contact between a sphere and a half-space where  $E_1=E_2/2=E_{sub}/4$ .

The capability of the semi-analytical method to solve complex contact problem between multi layered viscoelastic bodies is shown. The results of this modelling will allow a better understanding of the road behavior for example.

## 4. References

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