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Dissipation mechanisms and the crossover from Stokes to Coulomb friction

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Many different mechanisms are responsible for the friction that counteracts the relative sliding motion of two solids in mechanical contact. They range from elastic instabilities via plastic deformation to the flow of non-Newtonian fluids, i.e., boundary lubricants, to name a few. Most mechanisms have in common that they can be characterized as instabilities of certain (collective) degrees of freedom: an external force pushes part of the system to a maximum of the potential energy barrier, from where it quickly slides downhill into the next valley without giving the gained kinetic energy back to the solids' center-of-mass motion. In this contribution, I demonstrate that the velocity dependence of an isolated friction mechanism appears universal, i.e., Eyring like, at first sight. However, even a simple model like Prandtl's model exhibits a rate dependence of friction obeying the Carreau Yasuda equation (CYE). The latter applies to complex fluids like highly viscous polystyrene or camel blood. Particular attention is paid to the origin of the inaccuracy of the CYE at very small shear rates.

Keywords (from 3 to 5 max): dissipation, viscosity, instabilities, thermal activation, modeling

1. Introduction

Solid friction laws, in particular Coulomb's law (the kinetic friction force F_k depends only weakly on sliding velocity v) are intimately linked to instabilities [1]. The latter lead to finite kinetic friction in the limit of zero sliding velocities when thermal fluctuations are irrelevant. At finite temperature, F_k disappears linearly with v at $v \to 0$ and has a weak logarithmic or algebraic dependence (as already recognized by Coulomb) at intermediate v over several decades in velocity. A logarithmic dependence of $F_k$on v can be rationalized with Eyring's model and convincing comparisons be made to the shear-rate dependence of many polymers [2].

Simple models can reflect the just described behavior. A particularly successful example is the Prandtl model, which consists of a single atom that is pulled with a damped spring over a sinusoidal potential [1]. If the spring is sufficiently weak, Coulomb-like friction arises at intermediate velocities. The crossover from Stokes to Coulomb friction in the Prandtl model is surprisingly poorly investigated. The established view (the velocity dependence between the Stokes and Coulomb regime is close to logarithmic or a power thereof) must be challenged [3].

2. Methods

We study simple model systems with a focus on the Prandtl model using Brownian and Langevin dynamics as described in Ref. [3]. The transition from the discrete Prandtl model to a liquid model is made by assigning a height to a layer of ``Prandtl'' atoms.

3. Results and discussion

Figure 1 shows examples for the rate dependence of the effective viscosity η of the Prandtl model. For a large velocity range, it can be described with the Carreau Yasuda (CY) equation

$$\frac{\eta}{\eta_0} = \left\{1 + \left(\frac{\dot{\gamma}}{\gamma_0}\right)^a\right\}^{(n-1)/a}, \quad (1)$$

where η_0 is the equilibrium viscosity and $0 < a \leq 2$ and $0 < n < 1$ are exponents characteristic for a given liquid. In the Prandtl model, the CY exponents can be tuned through the ratio of spring stiffness and maximum substrate curvature, $\tilde{k}$, and the ratio of thermal energy and the amplitude of the substrate corrugation, $k_B\tilde{T}$.

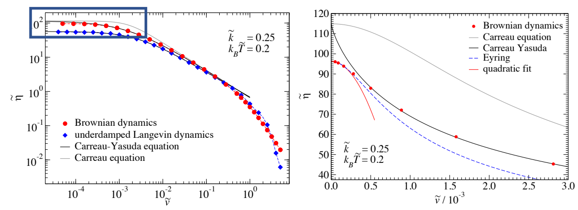

Figure 1: Viscosity as a function of effective shear rate of a simple model system and comparison to the Carreau and the Carreau-Yasuda equation. The *left panel* shows a large velocity regime in a double-logarithmic representation, while the *right panel* focuses on the small rates emphasized in the rectangle. From Ref. [3].

The original Carreau model arises as a limiting case of CY at $a = 2$. This value is expected from a perturbation-theory based expansion of F_k in v, which only allows even exponents in v. This feature might explain the popularity of the Carreau model. Nonetheless the presented analysis reveals that Carreau cannot be used to reliably deduce $\eta_0$from measurements of the viscosity in the Stokes-Coulomb crossover regime. Even CY can easily lead to errors exceeding 10\%, as demonstrated in the fit shown in Fig. 1.

4. References

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