Bearings with Stochastic Roughness on the Stator and the Rotor: Calculation of Higher-Order Moments of Operational Parameters

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In the framework of stochastic representation of surface roughness, a novel Reynolds-type Equation has been derived recently by the authors, and utilized to calculate the average values of a bearing's operational parameters. To acquire an improved insight to the hydrodynamic lubrication phenomena in rough bearings, the stochasticity of the operational parameters should be described, by calculating their higher-order moments, such as the variation. This work aims at deriving equations for calculation and quantification of these higher-order moments. The obtained results are validated against the Deterministic Reynolds Equation solution, for a series of bearings with different Gaussian surface roughness profiles.

Keywords: Stochastic Reynolds-type Equation, Finite Difference method, Stein's Lemma, Taylor's Expansion Theorem for moments of functions of random variables, Variation of Bearing Operational Parameters

1. Introduction

The effect of surface roughness in lubrication phenomena has long been a subject of intensive studies. In a previous work conducted by the authors [1], a novel stochastic Reynolds-type Equation was derived for hydrodynamically lubricated bearings with stochastic roughness on the stator and rotor, given by Equation (1).

$$\nabla\left(\begin{pmatrix} h^3 - 3\sigma^2 g^2(t) h \\ -3\sigma^2 g^2(t) h^2 \mathbb{F}_0 \\ +3\sigma^4 g^4(t)\mathbb{F}_0\end{pmatrix}\nabla m_p\right) = 12\eta\frac{\partial h}{\partial t} + 6U\eta\frac{\partial h}{\partial x} \quad \mathbf{(1)}$$

In Equation (1): h is the deterministic part of the stochastic film thickness, which is equal to its average value, and m_p the average value of the generated pressure. Equation (1), thus, imposes a relation between the first moments of the generated pressure and the stochastic film thickness. Similar equations, calculating the average value of the rest operational parameters of the bearing, are derived and presented in [1]. In order to be able to fully describe and gain a more holistic estimation on the effect of surface roughness in lubricated rough bearings, the determination of the stochasticity structure of its operational parameters' is needed.
The aim of the current work is to extend the results acquired by Equation (1) in order to derive equations for the calculation of the variance of a rough bearing's operational parameters.

2. Method

In the present work, utilizing the Stein's Lemma [2] and the Taylor's Expansion Theorem for moments of functions of random variables [3], we aim to calculate the variance of the generated pressure profile. The equations for the calculation of the rest operational parameters variance, i.e. the load carrying capacity, the friction force and the friction coefficient, are functions of the pressure's and film thickness mean value and variance.
The proposed equations are validated against deterministic models accounting for the surface roughness in the case of slider bearings. The validation is performed by utilizing the Finite Difference Method in the Deterministic Reynolds Equation (acquired by setting the roughness standard deviation in Equation (1) equal to zero) and generating random Gaussian surface roughness profiles. Histograms of the pressure variation, such as Figure 1, are created for a range of roughness deviation ratios and convergence ratios. Similar histograms are created for the rest of the operational parameters.

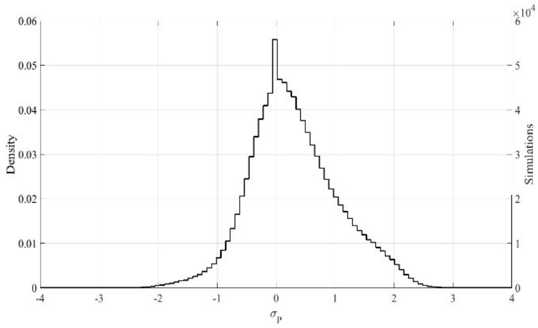

Figure 1: Simulations in slider bearing for convergence ratio $k = 1.5$, smooth rotor and stator ratio $r_1 = 0.2$

3. Discussion

It is noticed that as the simulation number is getting higher, most of the cases tend to calculate a pressure profile similar to the average generated pressure profile calculated by Equation (1). This is expected by the application of the *Law of Large Numbers (LLN)* of probability theory. Furthermore, a slight shift towards higher values of generated pressure is noticed.

4. References

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