Contact Stress Analysis of Rigid Indenter under Deep Penetration

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Deep penetration of a rigid indenter into a relatively soft and less stiff substrate is encountered in various engineering fields including geomechanics, biomedical engineering, and hydraulic fracking. Contact mechanics of deep penetration has received little attention as compared to surface indentation. The elastic solutions to deep penetration can be obtained by integrating the Mindlin solution to a point load applied at an interior point to the surface. Combining this approach with finite element analysis we arrive at the stress field around a spherical capped indenter. This solution agrees with the cone crack observed during needle penetration in hydrogels.

Keywords: Contact Stress, Deep Penetration, Fracture, Stress Trajectories

1. Introduction

Contact stress analysis of blunt rigid indenter has been well established on the surface indentation loading. However, the complex stress field around the rigid indenter in deep penetration has less attention. An analytical solution for a point load acting at an interior point in a semi-infinite solid was given by Mindlin [1]. Deep penetration with rigid blunt indenter is encountered in various engineering fields including geomechanics, biomedical engineering, and hydraulic fracking.

2. Methods

In this work, we study the deep penetration of rigid indenter loading at an interior point on a semi-infinite solid (see Fig 1). It is well known that surface indentation loading on the brittle materials with blunt indenter produces cone cracks [2]. The maximum principal stress in tensile just outside the contact zone is responsible for the nucleation of the crack. However, there has been no experimental evidence of cone crack that originate in the bulk of the material.

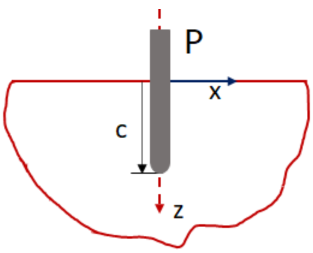

Figure 1: Schematic of rigid spherical tip indenter at a point interior in a semi-infinite solid.

3. Results and Discussion

The results show that the maximum principal stresses (σ_1) are tensile behind the indenter tip (see Fig 2 (a)). The cone crack nucleates at a point where the (σ_1) is maximum and crack propagates along with the minimum principal stress (σ_3) trajectory which is always perpendicular to the maximum principal stress trajectory (σ_1) (see Fig. 2 (b)). Figure 2 (c) shows the axisymmetric cone crack during deep penetration of the sharp rigid needle into soft hydrogel material [3].

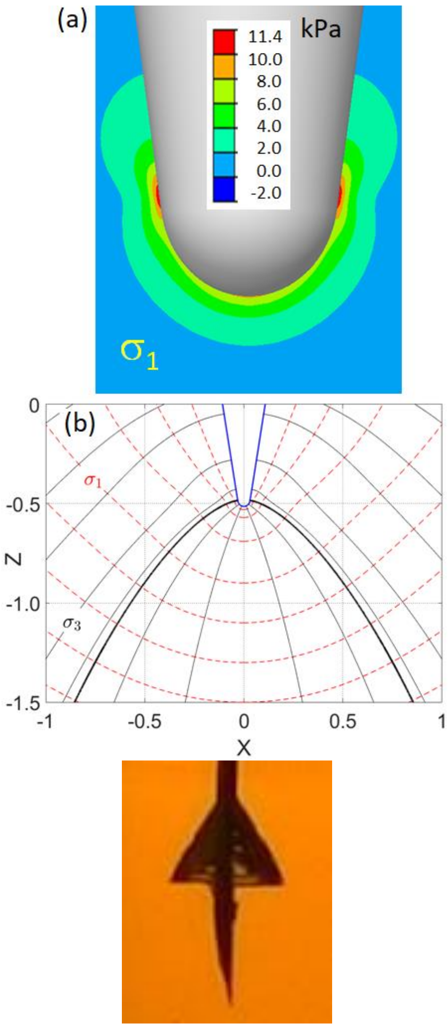

Figure 2: (a) Maximum principal stress (σ_1) contour. (b) Maximum (σ_1) and minimum (σ_3) principal stress trajectories. (c) Deep penetration of needle induced cone crack in a soft hydrogel.

4. References

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