

# How do 3D viscoelastic objects begin to slide?

## – Relation between precursor slip and friction coefficient –

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We numerically investigate the sliding motion of 3D viscoelastic objects. The maximum static friction coefficient depends on the pressure and the shape, which contradicts Amontons' law of friction. Our theoretical analysis clarifies the dependence of the friction coefficient results from precursor slip before macroscopic sliding.

**Keywords:** Amontons' law, viscoelastic object, precursor slip, friction coefficient, FEM calculation

### 1. Introduction

According to Amontons' law, the maximum static friction force of a solid object is proportional to the applied loading force and independent of the apparent contact area, which indicates that the static friction coefficient does not depend on the applied pressure and the shape of the solid. Previous studies have clarified that Amontons' law systematically breaks down for macroscopic objects due to precursor slip before the onset of bulk sliding [1], which is consistent with experiments [2]. However, the analysis in the previous works is restricted to 2D systems. Therefore, it is not clear whether the result is applicable to more realistic 3D systems.

### 2. 3D FEM calculation

To resolve the problems mentioned above, we numerically investigate the sliding motion of a 3D viscoelastic object with length  $L$ , width  $W$ , height  $H$ , and Young's modulus  $E$  on a solid substrate using the finite element method (FEM), as shown in Fig. 1. A rigid rod slowly pushes the object with a constant velocity. A uniform pressure  $P_{\text{ext}}$  is applied to the top of the object. The friction force at the bottom locally satisfies Amontons' law with the local static and kinetic friction coefficients  $\mu_S$  and  $\mu_K$ , respectively.

The object exhibits macroscopic stick-slip motions, which are characterized by the macroscopic static friction coefficient  $\mu_M$ .  $\mu_M$  decreases with the external pressure  $P_{\text{ext}}$  and the width  $W$ , as shown in Fig. 2, which contradicts Amontons' law. We also find that quasi-static precursor slip occurs in the 2D interface between the object and the substrate before bulk sliding, which is related to the decrease of  $\mu_M$ .

### 3. Analysis based on a 1D effective model

To theoretically understand the result of the 3D FEM simulation, we employ a 1D effective model, where the displacement is restricted in the driving direction and depends only on the position in the width direction. The model reproduces the results of the 3D FEM simulation semi-quantitatively. We clarify that the bulk sliding

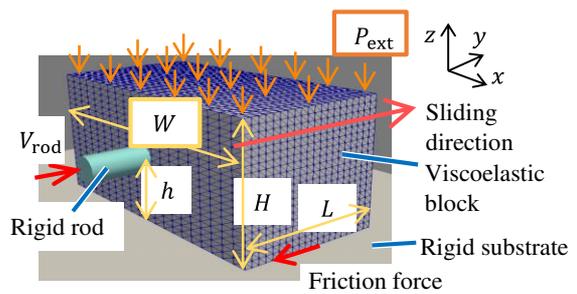


Figure 1: Schematic of the 3D FEM calculation.

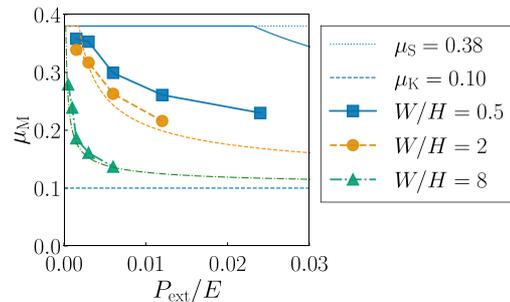


Figure 2: Macroscopic static friction coefficient  $\mu_M$  against  $P_{\text{ext}}$ . Filled symbols and thin lines represent the results of FEM calculation and the analysis based on the 1D effective model, respectively.

results from the instability of the quasi-static precursor slip which is caused by the velocity-weakening local friction. The critical size of the slip determines  $\mu_M$  and depends on  $P_{\text{ext}}$  and  $W$ , which explains the decrease of  $\mu_M$  in Fig. 2. The theoretical solutions are quantitatively consistent with the FEM results, especially for  $W/H = 2, 8$ , as shown in Fig. 2.

### 4. References

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- [2] Katano, Y. et al., "Novel Friction Law for the Static Friction Force based on Local Precursor Slipping," *Sci. Rep.*, 4, 2014, 6324.